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Black brane world from gravitating half σ-lump

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Abstract: We study O$(N+1)$ nonlinear σ-model in $(p+1+N)$-dimensional curved spacetime with a negative cosmological constant, and find a new σ-lump solution with half-integer winding and divergent energy. When the spatial structure of N extra-dimensions is determined by this global defect, a black σp-brane surrounded by the degenerated horizon is formed and its near-horizon geometry is identified as a warp geometry of a cigar type.

Keywords: Solitons Monopoles and Instantons, Extra Large Dimensions.

Contents

1. Introduction

From the early stage of brane world scenario [1] with warp geometry [2, 3], one of the intriguing issues has been a construction of the brane world with a thick brane. The popular method is to assume the bulk fields and to find a gravitating static defect solution which forms a thick brane in the extra-dimensions.

When the extra-dimension is one, a natural solitonic configuration is domain wall [4]. For two or more spatial extra-dimensions, rotational symmetry is assumed and then the corresponding internal symmetry was also continuous. Without a negative bulk cosmological constant, the global vortex in two extra-dimensions encounters a mild naked singularity [5] as has been done in (2+1)-dimensions [6]. Inclusion of a negative cosmological constant with the global vortex reproduces a singularity-free warp metric with a thick brane [7] as the extension from zero-dimensional brane [8]. The obtained intriguing thick brane world models include global and local vortices [7] and [9]–[12], global and local monopoles [13]–[16], topological lumps in nonlinear σ-model [17], and their higher dimensional analogues [9, 13, 14, 10, 18]. Their zero thickness limits are also studied [19].

The familiar brane world metric obtained in the above is summarized by

$$ds^2 = e^{2A(R)}(dt^2 - dx^{i2}) - dR^2 - C^2(R)d\Omega_{N-1}^2\,. \tag{1.1}$$

Here the warp factor $A(R)$ of interest is usually proportional to $-R$, $C(R)$ is either a constant for cigar geometry or an exponential form, and the angular part $d\Omega_{N-1}^2$ has solid deficit angle due to the global defects with a long range energy tail. If we look into the first two terms in the warp metric (1.1), an interesting structure of a degenerated horizon at $r_{\rm H} \neq 0$ arises through a transformation to Poincaré coordinates

$$ds^2 = (r-r_{\rm H})^2(dt^2 - dx^{i2}) - \frac{dr^2}{(r-r_{\rm H})^2} - C^2(R(r))d\Omega_{N-1}^2\,. \tag{1.2}$$

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This black brane structure is materialized by the global defects [10] irrespective of the dimensions of the black brane [8]. About a possible source of this horizon structure, it seems likely to be the divergent energy of global defects, which is not so harmful in the brane world model because of the negative vacuum energy proportional to the spatial volume of extra-dimensions at each point on the brane. An appropriate model for settling this question is nonlinear σ-model. In (2+1)-dimensional anti-de Sitter spacetime, O(3) nonlinear σ-model is known to support a half σ-lump with divergent energy in addition to a topological σ-lump with finite energy [20]. The half σ-lump can form an extremal charged BTZ black hole but the topological σ-lump cannot in (2+1)-dimensions.

In this paper, we will examine $\mathrm{O}(N+1)$ nonlinear σ-model in $(p+1+N)$-dimensional bulk with a flat p-brane and N spatial extra-dimensions. In ref. [17], the topological σ-lump was studied and the obtained warp geometry does not show the structure of black brane world as expected. We will show that there exists the half σ-lump and find the corresponding degenerated horizon where a cigar type geometry with an exponentially-decaying warp factor is realized.

The rest of the paper is organized as follows. In sections 2 and 3, we consider $\mathrm{O}(N+1)$ nonlinear σ-model in $(D=p+1+N)$-dimensional bulk and find the nontopological half σ-lump solution of which warp geometry is obtained near-degenerated-horizon. In section 4, we study in detail the properties of the warp geometry and its horizon. We conclude with summary and discussion in section 5.

2. $\mathrm{O}(N+1)$ nonlinear σ model in $(p+1+N)$-dimensions

We begin this section with introducing $\mathrm{O}(N+1)$ nonlinear σ-model in D-dimensional curved spacetime described by the action

$$S=\int d^D x\sqrt{(-1)^{D+1}g_D}\left[-\frac{M_*^{D-2}}{16\pi}(R+2\tilde{\Lambda})+\frac{1}{2}g^{AB}\nabla_A\phi^I\nabla_B\phi^I-\frac{\lambda(x)}{2}(\phi^J\phi^J-v^2)\right], \tag{2.1}$$

where $A,B,\ldots,$ denote D-dimensional bulk indices and internal indices, $I,J,\ldots,$ run from 1 to $N+1$. Note that both scalar field $\phi^I(x)$ and vacuum expectation value v have the same mass dimension $(D-2)/2$, and an auxiliary field $\lambda(x)$ mass dimension two. M_* is the mass scale of D-dimensional gravity and bulk cosmological constant $\tilde{\Lambda}$ has mass dimension two, which is assumed to be negative.

Euler-Lagrange equation for the scalar field after eliminating the auxiliary field $\lambda(x)$ is

$$\Box\phi^I-\left(\phi^J\,\frac{\Box}{v^2}\,\phi^J\right)\phi^I=0\,. \tag{2.2}$$

Einstein equations are given by

$$R^A{}_B=\frac{8\pi}{M_*^{p+1}}\mathcal{T}^A{}_B-\frac{2\tilde{\Lambda}}{p+N-1}\delta^A{}_B\,, \tag{2.3}$$

where energy-momentum tensor T^{AB} reads

$$T^{AB}=\nabla^A\phi^I\nabla^B\phi^I-g^{AB}\left[\frac{1}{2}g^{CD}\nabla_C\phi^I\nabla_D\phi^I-\frac{\lambda(x)}{2}\left(\phi^I\phi^I-v^2\right)\right], \tag{2.4}$$

and

$$T \equiv T^{M}{}_{M}, \qquad \mathcal{T}^{A}{}_{B} \equiv T^{A}{}_{B} - \frac{T}{p+N-1}\delta^{A}{}_{B}\,. \tag{2.5}$$

We are interested in the defect configurations of which spatial transverse dimension is p. Nontrivial winding exists between N-dimensional extra-dimensions and internal N-dimensional sphere of $\phi^I\phi^I = v^2$. Thus, the dimension of bulk spacetime is given by $D = p+N+1$ and we will call such defect as a *σ-lump* in the context of soliton solution or a *σp-brane* for the description of brane world from here on since the defect does not depend on the p-transverse spatial coordinates $\{x^i|i = 1, 2, \cdots, p\}$. The rotationally-symmetric defects are supposed to live in the extra-dimensions so that an appropriate choice of metric is Poincaré type coordinates:

$$ds^2 = B(r)e^{2\Phi(r)}(dt^2 - dx^{i2}) - \frac{dr^2}{B(r)} - r^2 d\Omega^2_{N-1}\,, \tag{2.6}$$

where $d\Omega^2_{N-1}$ denotes solid angle in the N extra-dimensions : $d\Omega^2_{N-1} = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{N-2} d\theta^2_{N-1}$ in the spherical coordinates $(r, \theta_1, \theta_2, \cdots, \theta_{N-1})$. We will show in the next section that the brane world of our interest is obtained either when it is bounded by a degenerated Cauchy horizon of $p+N-1$ dimensions or when the asymptotic region of sufficiently large distance is considered. Therefore, in what follows, we shall also call the horizon as *black brane horizon* and the $(p+N+1)$-dimensional bulk spacetime as *black brane world.*

For static defect solutions, we take a hedgehog ansatz

$$\phi^I = v\hat{\phi}^I\,, \tag{2.7}$$

where

$$\begin{aligned}\hat{\phi}^I = \;& (\sin F(r)\sin n\theta_1 \cdots \sin n\theta_{N-2}\sin n\theta_{N-1}, \sin F(r)\sin n\theta_1 \cdots \sin n\theta_{N-2}\cos n\theta_{N-1},\\ & \sin F(r)\sin n\theta_1 \cdots \sin n\theta_{N-3}\cos n\theta_{N-2}, \ldots, \sin F(r)\sin n\theta_1 \cos n\theta_2,\\ & \sin F(r)\cos n\theta_1, \cos F(r))\,.\end{aligned}$$

In order to keep the rotational symmetry in the extra-dimensions, n is an arbitrary natural number for $N = 2$ but should be unity for $N \geq 3$.

For the metric (2.6) and the ansatz (2.7), the scalar field equation (2.2) becomes

$$BF'' + \left[\frac{p+2}{2}B' + (p+1)B\Phi' + \frac{N-1}{r}B\right]F' - (N-1)\frac{n^2}{r^2}\sin F\cos F = 0\,, \tag{2.8}$$

where prime $'$ denotes derivative of r. The Einstein equations (2.3) have the following nonvanishing components

$$\begin{aligned}R^t{}_t =\;& \frac{B''}{2} + B\Phi'' + \frac{p}{4}\frac{B'^2}{B} + \frac{2p+3}{2}B'\Phi' + (p+1)B\Phi'^2 + \frac{N-1}{2}\frac{B'}{r} + (N-1)\frac{B\Phi'}{r}\\ =\;& 8\pi G_D \mathcal{T}^t{}_t - \frac{2\tilde{\Lambda}}{p+N-1}\,,\end{aligned} \tag{2.9}$$

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$$
\begin{aligned}
R^r{}_r &= (p+1)\left(\frac{B''}{2}+B\Phi''+\frac{3}{2}B'\Phi'+B\Phi'^2\right)+\frac{N-1}{2}\frac{B'}{r}\\
&= 8\pi G_D \mathcal{T}^r{}_r-\frac{2\tilde{\Lambda}}{p+N-1}\,, && (2.10)\\
R^{\theta_a}{}_{\theta_a} &= \frac{p+2}{2}\frac{B'}{r}+(p+1)\frac{B\Phi'}{r}+(N-2)\frac{B-1}{r^2}\\
&= 8\pi G_D \mathcal{T}^{\theta_a}{}_{\theta_a}-\frac{2\tilde{\Lambda}}{p+N-1}\,, && (2.11)
\end{aligned}
$$

where nonvanishing components of the energy-momentum tensor (2.5) are

$$
\mathcal{T}^t{}_t = \mathcal{T}^i{}_i = -\frac{N-2}{2(p+1)}\left[BF'^2+(N-1)\frac{n^2}{r^2}\sin^2 F\right], \tag{2.12}
$$

$$
\mathcal{T}^r{}_r = -\frac{N-2}{2(p+1)}\left[\frac{2p+N}{N-2}BF'^2+(N-1)\frac{n^2}{r^2}\sin^2 F\right], \tag{2.13}
$$

$$
\mathcal{T}^{\theta_a}{}_{\theta_a} = -\frac{N-2}{2(p+1)}\left[BF'^2+\frac{(N-1)(N-2)+2(p+1)}{N-2}\frac{n^2}{r^2}\sin^2 F\right], \tag{2.14}
$$

and their trace in eq. (2.5) is

$$
T = \frac{p+N-1}{2}\left(BF'^2+\frac{N-1}{2}\frac{n^2}{r^2}\sin^2 F\right). \tag{2.15}
$$

In the above we have used rescaled variables for both the coordinates and the cosmological constant as $x^A(\equiv v^{2/(D-2)}x^A)$ and $\Lambda(\equiv \tilde{\Lambda}/v^{4/(D-2)})$. Dimensionless Newton's constant is also defined by $G_D \equiv v^2/M_*{}^{D-2}$. The right-hand sides of the Einstein equations (2.9)–(2.11) involve three types of energy-momentum contributions, i.e., they are short-range derivative of the scalar amplitude F, solitonic long-range term proportional to n^2, and negative vacuum energy Λ proportional to the volume of the bulk.

Independent components of the Einstein equations are summarized by two nonlinear equations including first-order differential terms. Specifically, they are B, B', and Φ' in addition to F and F': One is the $\theta_a\theta_a$-component (2.11) and the other is obtained by subtraction of the rr-component multiplied by $p+1$ from the tt-component (2.9)

$$
\begin{aligned}
&\frac{1}{4}\frac{B'^2}{B}+B'\Phi'+B\Phi'^2+\frac{N-1}{r}\left[\frac{1}{2(p+1)}B'+\frac{1}{p}B\Phi'\right]= && (2.16)\\
&= 8\pi G_D\frac{N-2}{2(p+1)^2}\left[\frac{2(p+1)-p(N-2)}{p(N-2)}BF'^2-(N-1)\frac{n^2}{r^2}\sin^2 F\right]-\frac{2\Lambda}{(p+1)(p+N-1)}\,.
\end{aligned}
$$

In the case of two extra-dimensions ($N=2$), one may suspect possibility of self-duality of O(3) nonlinear σ-model, described by the first-order equation. Once the cosmological constant is turned on, it has been proved that any static soliton configuration of the first-order self-dual equation does not satisfy the second-order Euler-Lagrange equation (2.2) under any static metric even for the $p=0$ brane case [20]. In fact, for the $p=0$ brane of O(3) nonlinear σ-model, self-duality can be saturated only under the stationary metric and the obtained self-dual topological lump solution constitutes a spacetime with closed time-like curves [21]. It means that there does not exist any Bogomolnyi type bound in this brane world irrespective of p and N.

3. Half σ-lump as a σp-brane

In this section, by examining the equations of motion, we will find gravitating static half σ-lump solution of which nature is nontopological. Spacetime structure formed by the half lump configuration involves an extremal charged black σp-brane with a single degenerated horizon.

Boundary conditions at the origin, $r = 0$, are determined by nonsingularity of the fields and the metric functions:

$$F(0) = 0\,, \qquad B(0) = 1\,, \qquad \Phi(0) = \Phi_0\,, \tag{3.1}$$

where Φ_0 is always set to be zero by reparametrization of the spacetime variables of the p-brane. Near the origin, the series solutions up to leading term are

$$F(r) \approx F_0 r^n + \cdots\,, \tag{3.2}$$

$$B(r) \approx 1 - \frac{2}{N-1}\left[\frac{p-N+1}{N(p+N-1)}|\Lambda| + \frac{2(p+1)-(N-2)(p-N+1)}{(p+1)} \times \right.$$
$$\left. \times\, 2\pi G_D n^2 {F_0}^2 \delta_{n1}\right] r^2 + \cdots\,, \tag{3.3}$$

$$\Phi(r) \approx \Phi_0 + \frac{1}{N-1}\left[\frac{p}{N(p+N-1)}|\Lambda| + \frac{2(p+1)-p(N-2)}{p+1} \times \right.$$
$$\left. \times\, 2\pi G_D n^2 {F_0}^2 \delta_{n1}\right] r^2 + \cdots\,, \tag{3.4}$$

$$B(r)e^{2[\Phi(r)-\Phi_0]} \approx 1 + 2\left[\frac{1}{N(p+N-1)}|\Lambda| + \frac{N-2}{p+1} 2\pi G_D n^2 {F_0}^2 \delta_{n1}\right] r^2 + \cdots\,, \tag{3.5}$$

where F_0 is an undetermined constant decided by proper behavior of the scalar field at opposite boundary, e.g., spatial infinity at $r = \infty$ or a horizon at $r = r_{\rm H}$ if it exists.

Since the metric function Φ itself does not appear in the Einstein equations (2.11)–(2.16), those equations are understood as a first-order differential equation for $B(r)$ and an algebraic equation for $\Phi'(r)$ with initial values, i.e., the boundary conditions are $B(0) = 1$ or $\Phi'(0) = 0$. It means that there is only one free parameter F_0 in the series solutions (3.2)–(3.4) to adjust for the existence and location of the horizon, or equivalently the minimum value of $B(r)$, so that expected structure of the horizon is determined by a fine-tuning of the parameter F_0 for given M_*, $|\Lambda|$, and $\sqrt{\lambda}v$. Since $B(r)e^{2[\Phi(r)-\Phi_0]}$ in eq. (3.5) is always increasing near the origin, one cannot get some useful information on the existence of the horizon from the series expansion near the origin. As confirmed by the numerical work later, the metric $B(r)$ is likely to be decreasing near the origin. A plausible condition to get a horizon $r_{\rm H}$ specified by $B(r_{\rm H}) = 0$ is to make $B(r)$ a decreasing function near the origin, i.e., the conditions are $(dB/dr)|_{r=0} = 0$ and $(d^2B/dr^2)|_{r=0} < 0$. Since the metric function $B(r)$ is convex up around its minimum point, the position r_- of vanishing $B(r)$ in eq. (3.3) is smaller than the horizon

$$r_- = \left\{\frac{2}{N-1}\left[\frac{p-N+1}{N(p+N-1)}|\Lambda| + \frac{2(p+1)-(N-2)(p-N+1)}{p+1} 2\pi n^2 G_D F_0^2 \delta_{1n}\right]\right\}^{-1/2}$$
$$< r_{\rm H}\,. \tag{3.6}$$

When the metric function $B(r)$ has the minimum at $r = r_{\rm min}$, one can classify the cases into three by signature of the value of $B(r_{\rm min})$, i.e., $B(r_{\rm min}) > 0$, $=0$, or < 0. Above all let us take into account $B(r_{\rm min}) = 0$ case which implies coincidence of the horizon $r_{\rm H}$ and the minimum $r_{\rm min}$. The reason why is the formation of a brane world of an exponentially-decaying warp factor is constituted in the case of $B(r_{\rm H}) = 0$ as we will show. Our basic assumption is finiteness of the field $F(r)$ and its derivative $F'(r)$ at the horizon $r_{\rm H}$, which reflects nonsingular nature of the configuration of our interest. Then it is natural to assume vanishing of both the metric B and its derivative B' at the horizon as $B(r_{\rm H}) = B'(r_{\rm H}) = 0$. The resultant leading behavior of the metric function $B(r)$ is

$$B(r) \approx B_{\rm H}(r_{\rm H} - r)^2 + B_s(r_{\rm H} - r)^{2+s} + \cdots\,, \tag{3.7}$$

where $s \geq 1$. Since the metric function $\Phi(r)$ appears only as its exponential in the metric or derivative of it, a logarithmic singularity of Φ can be harmless for an appropriate value of α including $\alpha = 0$:

$$e^{2[\Phi(r) - \Phi_{\rm H}]} \approx (r_{\rm H} - r)^{-2\alpha}\,, \tag{3.8}$$

where $\Phi_{\rm H}$ is another undetermined constant. Later, for arbitrary α, we will find an appropriate coordinate transformation. In the scalar equation (2.8), all the terms should vanish at the horizon except for the last trigonometric function term due to the behavior of the metric functions (3.7)–(3.8). It asks vanishing of the last term such that the scalar amplitude should reach one of the following boundary value at the horizon $r_{\rm H}$:

$$F(r_{\rm H}) = \begin{cases} m\pi & \text{from the sine term,} \\ \left(m + \dfrac{1}{2}\right)\pi & \text{from the cosine term.} \end{cases} \tag{3.9}$$

Inserting the expansions (3.7)–(3.8) with the boundary condition of the scalar field (3.9) into the $\theta_a\theta_a$-component (2.11) of the Einstein equations, we express boundary value of the scalar amplitude F at the horizon as

$$\sin F(r_{\rm H}) = \pm\sqrt{\frac{1}{8\pi G_D n^2}\left[\frac{2(p+1)}{2(p+1) + (N-1)(N-2)}\right]\left[(N-2) + \frac{2|\Lambda|}{p+N-1}r_{\rm H}^2\right]}\,. \tag{3.10}$$

For the topological σ-lump with $F(r_{\rm H}) = \pi$, a mismatch in eq. (3.10) prohibits any nontrivial solution as far as the cosmological constant Λ is negative. On the other hand, for the nontopological σ-lump of half-winding, we can express the location of the horizon $r_{\rm H}$ in terms of the parameters of the theory

$$\begin{aligned} r_{\rm H} &= \sqrt{\frac{p+N-1}{2|\Lambda|}\left\{8\pi G_D n^2\left[1 + \frac{(N-1)(N-2)}{2(p+1)}\right] - (N-2)\right\}} \\ &\overset{N=2}{\Longrightarrow} \sqrt{\frac{4(p+1)\pi G_D}{|\Lambda|}}\,n\,. \end{aligned} \tag{3.11}$$

As shown in the case of two extra-dimensions ($N = 2$), an intriguing but expected property is the fact that radius of the horizon $r_{\rm H}$ is proportional to the vorticity n which is a dimensionless parameter representing popularity of overlapped σ-lumps.

Positivity of the horizon $r_{\rm H}$ in eq. (3.11) requires a condition

$$8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right] > N-2\,, \tag{3.12}$$

and it means that the natural scale of bulk nonlinear σ-field is supermassive or Planck scale since the vorticity n is unity as far as N is larger than two. When $n=1$, comparison between eq. (3.6) and eq. (3.10) provides a restriction on the shooting parameter F_0 of eq. (3.2):

$$\begin{aligned} F_0 > &\left\{\frac{|\Lambda|}{4\pi G_D}\frac{2(p+1)}{(p+N-1)[2(p+1)-(N-2)(p-N+1)]}\times\right. \\ &\left.\times\left[\frac{(p+1)(N-1)}{4\pi G_D[2(p+1)+(N-1)(N-2)]-(p+1)(N-2)}-\frac{p-N+1}{N}\right]\right\}^{1/2}. \end{aligned} \tag{3.13}$$

When $p-N+1\le 0$, positivity of the square root in Eq. (3.13) asks supermassive symmetry breaking scale for any reasonable, finite N:

$$4\pi G_D > \frac{(p+1)(N-1)}{2(p+1)+(N-1)(N-2)}\,. \tag{3.14}$$

When $p-N+1>0$, it gives another range with an upper bound for the scale of the bulk nonlinear σ-field

$$\frac{(p+1)(N-2)}{2(p+1)+(N-1)(N-2)} < 4\pi G_D < \frac{(p+1)[N(N-1)+(N-2)(p-N+1)]}{(p-N+1)[2(p+1)+(N-1)(N-2)]}\,. \tag{3.15}$$

For multi-defects ($n>1$) in two extra-dimensions, we also have a similar condition by comparing eq. (3.6) to eq. (3.11) such as $4\pi G_D n^2(p-1)>1$. If sufficiently-large number of the half σ-lumps are superimposed, an intriguing observation is made under the crude approximation: the scale of spontaneous symmetry breaking $v^{2/(D-2)}$ needs not to be supermassive, and a possible hierarchy between the Planck scale M_* and the symmetry breaking scale $v^{2/(D-2)}$ is probably controlled by huge topological winding n.

Finally, leading term of the tt-component of the Einstein equations (2.9) and second-order expansion of the $\theta_a\theta_a$-component fix the coefficients of the metric functions (3.7)–(3.8) as

$$B_{\rm H} = \frac{8|\Lambda|}{(p+N-1)\left\{2+(p+1)^2\kappa\mp(p+1)\sqrt{[4+\kappa(p+2)^2]\kappa}\right\}}\,, \tag{3.16}$$

$$\alpha = 1-\frac{p+1}{2}\kappa\pm\sqrt{\left[1+\left(\frac{p+1}{2}\right)^2\kappa\right]\kappa}\,, \tag{3.17}$$

where

$$\kappa = \frac{1}{2(p+1)}-\frac{(p+1)(N-1)(N-2)}{2(p+1)+(N-1)(N-2)-\frac{(p+1)(N-2)}{4\pi G_D n^2}}\,. \tag{3.18}$$

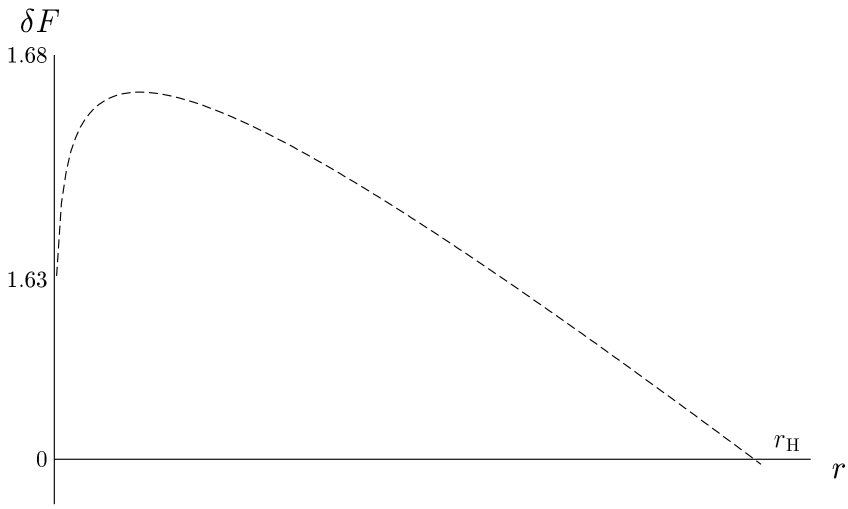

Figure 1: A representative shape of $\delta F(r)$ for a σ3-brane with three extra-dimensions when $|\Lambda| = 1$, $4\pi G_D = 1$, and $n = 1$. Coefficients, C_+ and C_-, are chosen as -19.6016 and 3.395, respectively. Aforementioned singularity appears at the horizon $r_{\rm H}$.

Furthermore, the power of the second term of B in eq. (3.7) is also fixed by one, $s = 1$. Therefore, the lapse function in front of time variable t results in

$$B(r)e^{2[\Phi(r)-\Phi_{\rm H}]} \approx \frac{8|\Lambda|}{(p+N-1)\left\{2+(p+1)^2\kappa \mp (p+1)\sqrt{[4+\kappa(p+2)^2]\kappa}\right\}} \times \\ \times [\sigma(r_{\rm H}-r)]^{\{(p+1)\kappa \mp \sqrt{[4+(p+1)^2\kappa]\kappa}\}}\,, \tag{3.19}$$

where $\sigma = +1$ for the interior region and $\sigma = -1$ for the exterior region. Between two possible solutions in eqs. (3.16)–(3.19), a necessary condition for vanishing $Be^{2(\Phi-\Phi_{\rm H})}$ at $r_{\rm H}$ selects the lower sign in eqs. (3.16)–(3.19) when κ is positive. To be specific, the condition of positive κ is rephrased by one of the following two possible conditions for $4\pi G_D n^2$: Either

$$4\pi G_D n^2 < \frac{(p+1)(N-2)}{2(p+1)+(N-1)(N-2)} \tag{3.20}$$

or

$$4\pi G_D n^2 > \frac{(p+1)(N-2)}{2(p+1)+(N-1)(N-2)-2(p+1)^2(N-1)(N-2)}$$

together with

$$\frac{2(p+1)}{2p^2+4p+1} > (N-1)(N-2)\,. \tag{3.21}$$

Comparison of eq. (3.20) with eqs. (3.14)–(3.15) tells us that the condition given by eq. (3.20) seems unlikely. Similarly, a probable condition is provided by the intersection of eq. (3.21) and eq. (3.14) or that of eq. (3.21) and eq. (3.15).

For a better understanding of leading behavior of the scalar field $F(r)$ near the horizon $r_{\rm H}$, we consider a linearized equation for $\delta F(r)$ defined by $F(r) = \pi/2 + \delta F(r)$. As an approximation of $B(r)$ and $\Phi(r)$ we bring up the series solutions (3.7)–(3.8). Substitution of these into the scalar equation (2.8) leads to a linear equation of δF

$$x^2 \frac{d^2\delta F}{dx^2} + [1+(p+1)(1-\alpha)]x\frac{d\delta F}{dx} - \delta(p+1)(1-\alpha)^2\delta F = 0\,, \qquad (3.22)$$

where $1/x \equiv r_{\rm H} - r$ and $\delta = (N-1)n^2/\kappa\{4\pi G_D n^2[2(p+1)+(N-1)(N-2)] - (p+1)\times(N-2)\}$. Note that the previous condition for eq. (3.21) asks δ to be positive. Then an exact solution of eq. (3.22) is

$$\delta F(x) = C_+ x^{k_+} + C_- x^{k_-}\,, \qquad (3.23)$$

where $k_\pm = 1/4(p+1)^2\kappa[1+\sqrt{[1+(\frac{2}{p+1})^2\frac{1}{\kappa}]}][-1\pm\sqrt{1+\frac{4\delta}{p+1}}] > 0$, for both upper and lower signs, and $C_\pm$ are determined by proper behaviors of the scalar field at both boundaries, $x = r_{\rm H}$ and $x = 0$. A specific form of δF is shown in figure 1. Trying a power series solution $\delta F \approx F_1(r_{\rm H} - r)^t$ in the scalar equation (3.22), we cannot fix the coefficient F_1 but t becomes

$$\begin{aligned} t = {} & \frac{1}{4}(p+1)^2\kappa\left[1+\sqrt{1+(\frac{2}{p+1})^2\frac{1}{\kappa}}\right]\times \qquad (3.24) \\ & \times\left\{-1\pm\sqrt{1+\frac{2n^2(N-1)}{\kappa(p+1)\left\{8\pi G_D n^2[2(p+1)+(N-1)(N-2)]-2(p+1)(N-2)\right\}}}\right\}. \end{aligned}$$

Since the terms in the square root is positive, t is also positive for both upper and lower signs, which lets the perturbation allowable. In order to describe our world of $p = 3$, we have $t = 0.161$ for two extra-dimensions ($N = 2$) and $t = 0.019$ for three extra-dimensions ($N = 3$) in the Planck scale such as $4\pi G_D n^2 = 1$. The obtained values of t are not natural numbers so that $F(r)$ is mostly nonanalytic near the horizon but both $F(r_{\rm H})$ and $F^{'}(r_{\rm H})$ are finite as expected.

Since second-order expansion of the Einstein equations (2.9)–(2.11) does not involve contribution of the scalar field, the coefficients of second-order terms in eqs. (3.7)–(3.8) are given as

$$\begin{aligned} B_s = {} & \frac{2}{(p+1)\alpha+(p+4)}\left\{\frac{p+N-1}{2|\Lambda|}\times\right. \\ & \left.\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}\right\}^{-1/2}\times \\ & \times\left\{\frac{8|\Lambda|}{(p+N-1)[2+(p+1)^2\kappa\mp(p+1)\sqrt{[4+\kappa(p+2)^2]\kappa}]}\right\}\times \\ & \times\left\{p[(p+1)\alpha-(p+2)]\sqrt{\frac{p+N-1}{2|\Lambda|}\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}}+\right. \\ & \left.+p(N-2)-\alpha(p+1)(N-1)\right\}, \qquad (3.25) \end{aligned}$$

$$\Phi_{\mathrm{H}} = \frac{2-\alpha}{(p+1)\alpha+(p+4)}\times$$
$$\times\left\{(\alpha-1)(p+1)-1+\left[\frac{p(N-2)-\alpha(p+1)(N-1)}{p}\right]\times\right.$$
$$\left.\times\left\{\frac{p+N-1}{2|\Lambda|}\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}\right\}^{-1/2}\right\}\times$$
$$\times\left\{\frac{8|\Lambda|}{(p+N-1)[2+(p+1)^2\kappa\mp(p+1)\sqrt{[4+\kappa(p+2)^2]\kappa}]}\right\}-$$
$$-\frac{\alpha(N-1)}{p}\left\{\frac{p+N-1}{2|\Lambda|}\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}\right\}^{-1/2} \tag{3.26}$$

where α is given in eq. (3.17). When r is larger than r_{H}, the scalar amplitude $F(r)$ has its boundary value, $F(r)=\pi/2$, as mentioned in eq. (3.9).

Though the scalar equation (2.8) is trivially satisfied outside the horizon, the right-hand side of the Einstein equation (2.11) tells us that at the exterior region the negative vacuum energy from the cosmological constant dominates and the long-range topological term is a subleading term proportional to $\mathcal{O}(r^{-2})$. It does not violate no hair conjecture because there is no distinction between the source of the topological term and the charge of an antisymmetric tensor field of rank $N-1$ for an observer in the exterior region [21, 20]. Systematic expansion at asymptotic region gives the case that the field and metric functions behave approximately as those of pure anti-de Sitter spacetime:

$$B(r) \sim \frac{2|\Lambda|}{(p+N)(p+N-1)}r^2-\frac{2p+N-1}{p(p+1)+(N-2)(2p+N-1)}\times$$
$$\times\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}, \tag{3.27}$$

$$\Phi(r)-\Phi_\infty \sim \frac{(p+N)(p+N-1)}{4[(p+1)+(N-2)(2p+N-1)]}\times$$
$$\times\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}\frac{1}{|\Lambda|}\frac{1}{r^2}, \tag{3.28}$$

$$B(r)e^{2[\Phi(r)-\Phi_\infty]} \sim \frac{2|\Lambda|}{(p+N)(p+N-1)}r^2-$$
$$-\left\{\frac{2p+N-1}{p(p+1)+(N-2)(2p+N-1)}-\right.$$
$$\left.-\frac{1}{2[(p+1)+(N-2)(2p+N-1)]}\right\}\times$$
$$\times\left\{8\pi G_D n^2\left[1+\frac{(N-1)(N-2)}{2(p+1)}\right]-(N-2)\right\}, \tag{3.29}$$

where Φ_∞ is determined by the proper boundary values of the metric functions Φ and B at the origin or at the horizon r_{H} if it exists. Since the leading term of the metric function Φ is a constant and the subleading term decays rapidly for sufficiently large r, geometry of the exterior region is mostly governed by the metric function B. The existence of a horizon requires necessarily negativity of the constant term of the metric B (3.27) so that we reproduce the condition of a supermassive scale for $N > 2$ in eq. (3.12) obtained at the interior region of the horizon. Another rough estimation of the radius of the horizon can be performed from $e^{2\Phi(r_+)}B(r_+) = 0$ in eq. (3.28) or $B(\tilde{r}_+) = 0$ in eq. (3.27):

$$\begin{aligned}
\tilde{r}_+ &= \sqrt{\frac{(p+N)(2p+N-1)}{p(p+1)+(N-2)(2p+N-1)}}\, r_{\rm H} \\
&> r_+ = \sqrt{\frac{(p+N)(2p+N-1)}{p(p+1)+(N-2)(2p+N-1)} - \frac{p+N}{2[(p+1)+(N-2)(2p+N-1)]}}\, r_{\rm H} \\
&> r_{\rm H}\,. \qquad (3.30)
\end{aligned}$$

Since both prefactors in front of $r_{\rm H}$ in eq. (3.30) are a few for every N ($N \geq 2$), we read validity of the above expansion and signature of the next order term in the expansion should be negative. Note that $\lim_{N\to\infty} \tilde{r}_+ = \lim_{N\to\infty} r_+ = r_{\rm H}$.

Though the scalar field $F(r)$ and the metric function Φ are nonanalytic at the horizon $r_{\rm H}$, it does not mean existence of a physical singularity at the horizon. It can be proven by examining Kretschmann scalar $R^{ABCD}R_{ABCD}$. Computation of the Kretschmann scalar for the metric (2.6) is

$$\begin{aligned}
&R^{ABCD}R_{ABCD} = \\
&= +(p+1)(B''^2+6B''B'\Phi'+4B''B\Phi''+4B''B\Phi'^2+12B'B\Phi'\Phi'' + 4B^2\Phi''^2 + 8B^2\Phi'') + \\
&\quad +p(p+1)\left(\frac{B'^4}{8B^2}+\frac{B'^3\Phi'}{B}\right)+(p+1)(p+3)(3B'^2\Phi'^2+4BB'\Phi'^3)+3(p^2+p+1)B^2\Phi'^4+ \\
&\quad + \frac{N-1}{r^2}[(p+2)B'^2 + 4(p+1)(BB'\Phi' + B^2\Phi'^2)]\,. \qquad (3.31)
\end{aligned}$$

At the position of the σp-brane it has a finite value as expected :

$$\begin{aligned}
&R^{ABCD}R_{ABCD}\,|_{r=0} = \\
&= \frac{16(p+1)}{(N-1)^2}\Bigg\{\Bigg[\frac{p-N+1}{N(p+N-1)}|\Lambda| + \\
&\qquad + \frac{2(p+1)-(N-2)(p-N+1)}{(p+1)}2\pi G_D n^2 {F_0}^2\delta_{n1}\Bigg]^2 \times \\
&\qquad \times\left\{1+8\left[\frac{p}{N(p+N-1)}|\Lambda| + \frac{2(p+1)-p(N-2)}{p+1}2\pi G_D n^2{F_0}^2\delta_{n1}\right]\right\} - \\
&\qquad - \left[\frac{p}{N(p+N-1)}|\Lambda| + \frac{2(p+1)-p(N-2)}{p+1}2\pi G_D n^2{F_0}^2\delta_{n1}\right] \times \\
&\qquad \times\Bigg\{2\Bigg[\frac{p-N+1}{N(p+N-1)}|\Lambda| + \\
&\qquad\quad + \frac{2(p+1)-(N-2)(p-N+1)}{(p+1)}2\pi G_D n^2{F_0}^2\delta_{n1}\Bigg] - (N-1)\Bigg\} + \\
&\qquad + \left[\frac{p}{N(p+N-1)}|\Lambda| + \frac{2(p+1)-p(N-2)}{p+1}2\pi G_D n^2{F_0}^2\delta_{n1}\right]^2\Bigg\}, \qquad (3.32)
\end{aligned}$$

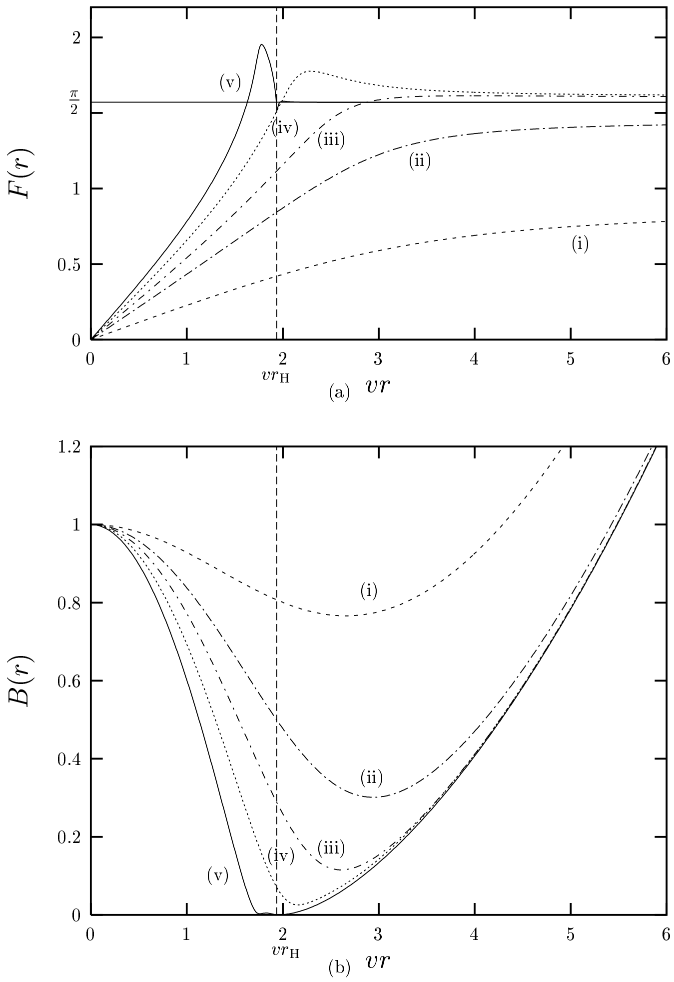

Figure 2: (a) Scalar amplitude $F(r)$ for various half-winding solitons when $p = 3$, $N = 3$, $8\pi G v^2 = 1.58$, $|\Lambda|/v^2 = 0.65$, and $n = 1$. The number of nodes increases as initial shooting parameter F_0 increases. (b) Plots of $B(r)$ show development of a horizon at $vr_{\rm H} = 1.94$ as the initial shooting parameter F_0 increases. For both figures, $F_0 = 0.23$ for (i), $F_0 = 0.43$ for (ii), $F_0 = 0.53$ for (iii), $F_0 = 0.63$ for (iv), $F_0 = 0.73$ for (v).

and substitution of eqs. (3.7) and (3.8) into eq. (3.31) gives

$$
\begin{aligned}
&R^{ABCD}R_{ABCD}\big|_{r=r_{\rm H}} = \\
&= \frac{4(p+1)\kappa|\Lambda|}{(p+N-1)[-2(p+1)+\kappa(p^2+4p+5)\mp(p+1)\sqrt{[4+\kappa(p+1)^2]\kappa}]}\times \\
&\quad\times\Big\{2(-p+3)+(16p^2-6p-25)\kappa+(p+1)(32p^2+13p-42)\kappa^2-\frac{1}{2}(8p+3)(p+1)^3\kappa^3\pm \\
&\qquad\pm 2[-2(4p-1)+p(24p+31)\kappa-(8p+3)(p+1)^2\kappa^2]\sqrt{[4+(p+1)^2\kappa]\kappa}\Big\}, \qquad (3.33)
\end{aligned}
$$

where κ is given in eq. (3.18). One can easily see that value of the Kretschmann scalar is finite at $r = r_{\rm H}$ and then the singularity at $r_{\rm H}$ is nothing but a coordinate singularity which

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also supports the claim that the position $r_{\rm H}$ is the location of the horizon of an extremal black hole. Furthermore, the terms in the expressions (3.31) and (3.32) tell us that the whole spacetime is free from physical singularity.

From now on let us consider the cases of nonvanishing $B(r_{\rm min})$. When $B(r_{\rm min}) > 0$, $\Phi(r)$ and $F(r)$ are smooth around $r_{\rm min}$ so that the bulk is smoothly connected to the pure anti-de Sitter space (See the lines (i)–(iv) in figure 2(b)). Therefore the leading behavior of the metric near $r \approx r_{\rm min}$ dictates flat spacetime.

When $B(r_{\rm min}) < 0$, there should exist at least two positions represented by $r_{\rm H}$ to make the metric function vanish such as $B(r_{\rm H}) = 0$. Then, the leading behavior of $B(r)$ is approximated by

$$B(r) \approx B_{\rm H}(r_{\rm H} - r) + \cdots . \tag{3.34}$$

Our basic assumption is the same as those of the extremal case such as finiteness of the scalar field $F(r)$ and its derivative, and mild allowable singular form of the metric function Φ in eq. (3.8). At any point $r_{\rm H}$ with vanishing $B(r)$, the Einstein equations (2.11) and (2.16) are summarized as

$$\left[BB'\Phi' + \frac{1}{4}B'^2 + (B\Phi')^2\right]\Bigg|_{r=r_{\rm H}} = 0\,, \tag{3.35}$$

$$\left[B^2\Phi'' + \frac{1}{2}BB'\Phi' - \frac{1}{4}B'^2\right]\Bigg|_{r=r_{\rm H}} = 0. \tag{3.36}$$

Since $B(r_{\rm H}) = 0$, both eqs. (3.35) and (3.36) require $B'(r_{\rm H}) = 0$ which forces impossibility of geometry with two horizons.

Throughout numerical computation, a prototype of σ3-brane solution with unit topological winding ($n = 1$) is displayed in figure 2. The extra-dimensions are three, the symmetry breaking scale is almost Planck scale ($8\pi G v^2 = 1.58$), and the absolute value of negative cosmological constant is also the square of the Planck scale ($|\Lambda|/v^2 = 0.65$). As shown in the solid line (v) of figure 2(a), the scalar amplitude starts from zero at the origin, $F(0) = 0$, and arrives at the boundary value of half-winding at the horizon, $F(r_{\rm H}) = \pi/2$ after several oscillations around the boundary value. In the exterior region, $r > r_{\rm H}$, the scalar amplitude remains at the boundary value. A horizon is developed at the position $r_{\rm H}$ as shown in the solid line of figure 2(b) by the half σ-lump configuration of $F(r_{\rm H}) = \pi/2$. Impossibility of the geometry with two horizons is also confirmed by numerical works.

In this section, we look for static defect of O$(N+1)$ nonlinear σ-model, which live in the N extra-dimensions with a negative cosmological constant. We find, in addition to usual topological σ-lump, a new half σ-lump solution of which energy is not finite. This divergent long-tail of the energy density is not harmful in anti-de Sitter spacetime but, intriguingly enough, it can form a degenerated horizon. In the next section, we will show that it is the horizon of an extremal black σp-brane and a coordinate transformation uncovers a near-horizon warp geometry of a black brane world.

4. Warp geometry near the horizon

When we completely neglected the effect of matter fields, the geometry is solely governed by the negative cosmological constant and, from eqs. (3.27)–(3.29), its metric is

$$d\tilde{s}^2 \approx \tilde{r}^2 d\tilde{x}^\mu d\tilde{x}_\mu - \frac{d\tilde{r}^2}{\tilde{r}^2} - \tilde{r}^2 d\tilde{\Omega}^2_{N-1}\,, \tag{4.1}$$

where we rescale the variables as $\tilde{s} = \sqrt{\Lambda_{\rm eff}}\, s$, $\tilde{r} = \Lambda_{\rm eff}\, r$, $\tilde{x}^\mu = e^{-\Phi_\infty} x^\mu$, $d\tilde{\Omega}^2_{N-1} = d\Omega^2_{N-1}/\Lambda_{\rm eff}$ and $\Lambda_{\rm eff} = 2|\Lambda|/(p+N)(p+N-1)$. Note that there is a solid deficit angle in extra-dimensions. A coordinate transformation to the familiar coordinate system (1.1), $\tilde{r} = e^{\pm\rho}(0 \le \rho \le \infty)$, leads to a warp geometry with an exponentially-varying circumference

$$d\tilde{s}^2 = e^{\pm 2\rho} d\tilde{x}^\mu d\tilde{x}_\mu - d\rho^2 - e^{\pm 2\rho} d\tilde{\Omega}^2_{N-1}. \tag{4.2}$$

Since we started without effect of the matter fields, the warp factor in eq. (4.2) should be universal independent of matter contents inside.

In what follows we interpret physical meaning of the near-horizon geometry obtained in the previous section. At the interior region of the degenerated horizon at $r_{\rm H}$, it is depicted by

$$ds^2 \approx \Lambda_{\rm H}(r_{\rm H} - r)^{\alpha_{\rm H}} dx^\mu dx_\mu - \frac{dr^2}{B_{\rm H}(r_{\rm H} - r)^2} - r^2 d\Omega^2_{N-1}\,, \tag{4.3}$$

where $r_{\rm H}, B_{\rm H}, \alpha_{\rm H}$, and $\Lambda_{\rm H}$ are given in eq. (3.11) and eq. (3.16)–eq. (3.19). Let us introduce a new radial variable $\bar{r}$ as $e^{\pm 2\bar{r}} = (r_{\rm H} - r)^{\alpha_{\rm H}}$ for small positive $e^{\pm 2\bar{r}}$ together with rescaling $\bar{s} = (\alpha_{\rm H}\sqrt{B_{\rm H}}/2)s$, $\bar{x}^\mu = (\alpha_{\rm H}\sqrt{\Lambda_{\rm H} B_{\rm H}}/2)x^\mu$, and $d\bar{\Omega}^2_{N-1} = (\alpha_{\rm H}{}^2 B_{\rm H}/4) d\Omega^2_{N-1}$. Here, the solid deficit angle at the horizon is due to the long tail of energy density of the half σp-lump. Then, the resultant metric describes a cigar type geometry with an exponentially decaying warp factor:

$$d\bar{s}^2 \approx e^{\pm 2\bar{r}} d\bar{x}^\mu d\bar{x}_\mu - d\bar{r}^2 - r^2_{\rm H} d\bar{\Omega}^2_{N-1}\,. \tag{4.4}$$

Simultaneously, another coordinate transformation $(R - r_{\rm H})^2 = e^{\pm 2\bar{r}}$ gives

$$d\bar{s}^2 \approx (R - r_{\rm H})^2 d\bar{x}^\mu d\bar{x}_\mu - \frac{dR^2}{(R - r_{\rm H})^2} - r^2_{\rm H} d\bar{\Omega}^2_{N-1}\,, \tag{4.5}$$

which shows clearly the degenerated horizon at $R = r_{\rm H}$ of an extremal black σp-brane.

We also study the detailed property of the black σp-brane world and demonstrate clearly that the obtained bulk spacetime near the horizon coincides exactly with the Randall-Sundrum type brane world with one σp-brane and N extra-dimensions. Since the spacetime structure of the σp-brane is flat and we assumed rotational symmetry in the N extra-dimensions, nontrivial part of the bulk geometry appears in the warp factor in front of the spacetime metric of the σp-brane and the radial component of the extra-dimensions as shown in eq. (1.2). For a given time, the spatial geometry of the extra-dimensions may mostly be depicted by the following quantities because of rotational symmetry of the extra-dimensions. They are the radial distance from the origin $r = 0$, $\mathcal{R}(r) = \displaystyle\int_0^r \frac{dr'}{\sqrt{B(r')}}$, the circumferential volume, $l(r) = \displaystyle\int d\Omega_{N-1} r^{N-1}$, and the spatial volume of the interior region of the bulk per unit volume of the σp-brane, $\mathcal{V}(r) = \int d\Omega_{N-1} \int_0^r dr' r'^{N-1} e^{(p+1)\Phi(r')} B(r')^{p/2}$. If we use the volume of an $(N-1)$ sphere of unit radius, $\int d\Omega_{N-1} = 2\pi^{N/2}/\Gamma(N/2)$, then we have $\int d\bar{\Omega}_{N-1} = [2\pi^{N/2}/\Gamma(N/2)] \times [\alpha_{\rm H}\sqrt{B_{\rm H}}/2]^{N-1}$, where the second factor stands for existence of a solid deficit angle due to the global defect sitting at the origin. For spatial hypersurface of $p+N$ dimensions, the radial distance from the origin $r=0$ to $r_{\rm H}$ is logarithmically divergent:

$$\mathcal{R}(r_{\rm H}) = \int_0^{r_{\rm H}} \frac{dr'}{\sqrt{B(r')}} \sim -\lim_{r\to r_{\rm H}} \ln|r_{\rm H} - r| \to \infty\,, \tag{4.6}$$

but the circumference is finite :

$$l(r_{\rm H}) = \frac{2\pi^{N/2}}{\Gamma(N/2)} \left(\frac{\alpha_{\rm H}}{2}\sqrt{B_{\rm H}} r_{\rm H}\right)^{N-1}\,. \tag{4.7}$$

Despite of the infinite radial distance and the finite circumferential volume, the spatial volume per unit σp-brane volume bounded by the horizon is finite due to the warp factor in front of the σp-brane metric (1.2):

$$\mathcal{V}(r_{\rm H}) = \frac{2\pi^{N/2}}{\Gamma(N/2)} \int d^p x \int_0^{r_{\rm H}} dr' r'^{N-1} e^{(p+1)\Phi(r')} B(r')^{p/2} \Big/ \int d^p x \sim \text{finite}. \tag{4.8}$$

For the case of two extra-dimensions ($N=2$), the value of $r_{\rm H}$ is proportional to the winding number n in eq. (3.11) so that the volume (4.8) can be large as n becomes large. It suggests that our black σp-brane world interpolates Randall-Sundrum type with two branes for small n to Arkani Hamed-Dimopoulos-Dvali type for sufficiently large n. Therefore, a geometric fine-tuning by a large extra-dimensions can be replaced by that of large population of half σp-lumps.

5. Summary

We studied $\mathrm{O}(N+1)$ nonlinear σ-model in $(p+1+N)$-dimensional curved spacetime with a negative cosmological constant. In addition to the well-known topological σ-lump solution, we found half σ-lump, another class of solution similar to the global defect in linear σ-model. When spatial N extra-dimensions are formed by this half σ-lump, our flat p-brane is located at the center of the global defect surrounded by the degenerated horizon of the black σp-brane and its near-horizon geometry is a warp geometry of cigar type (See figure 3). Existence of this half σ-lump can be detected by a solid deficit angle at the horizon due to its divergent energy. At the asymptotic region of anti-de Sitter space, another warp geometry is generated as usual (See figure 3).

Final comments are in order. Since the obtained horizon structure is extremal, we are temped to claim that the warp geometry near the degenerated horizon is a sort of energetically-favored configurations according to the experience in four-dimensional Reissner-Nordstrom black hole. However, justification of this statement should be postponed until we have complicated stability analysis as in ref. [22, 23]. Though we found

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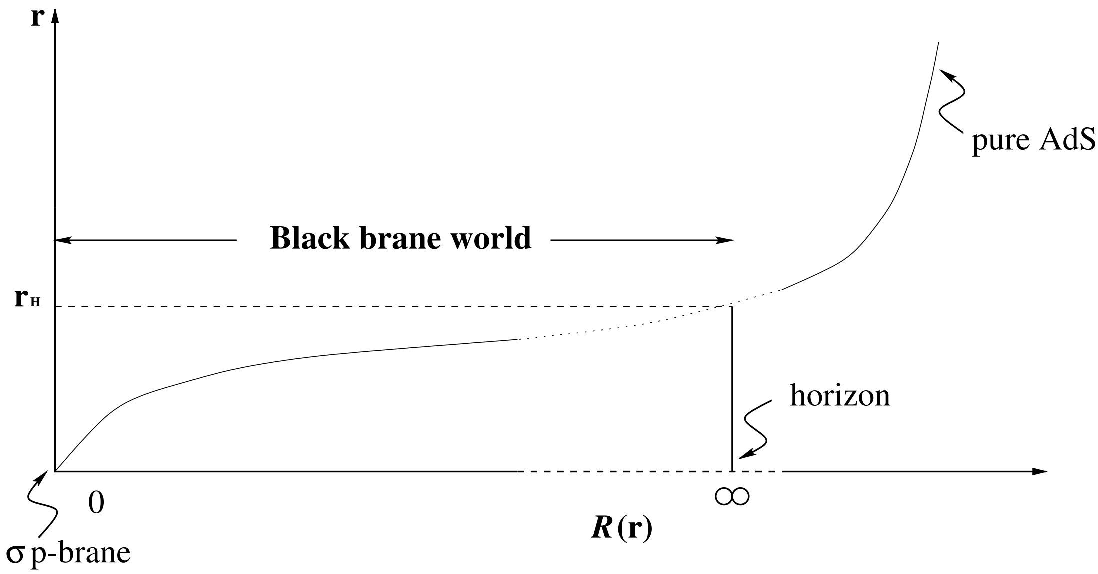

Figure 3: Schematic shape of a black σp-brane world.

both static topological and half σ-lump solutions, they are not BPS objects in anit-de Sitter spacetime and a lesson from (2+1)-dimensions is that the BPS lump is given by a stationary solution with closed timelike curves [21]. Recently, new global defect solutions have been found [24]. It may be intriguing to test the above warp structure by employing those objects.

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